MTH 201
Multivariable calculus and differential equations
Homework 4
Differentiation

1. Find all partial derivatives at $(0,0)$ (if exist) for each of the following function
   (a) $f(x,y) = e^{xy}\sin(x^2+y^2)$
   (b) $f(x,y) = \frac{\sin x}{1+y^2}$
   (c) $f(x,y) = \frac{x^2-y^2}{x^2+y^2}$, $(x,y) \neq (0,0)$ and $f(0,0) = 0$ (HW)
   (d) $f(x,y) = \frac{x^2\sin^2 y + y^2 \sin^2 x}{x^2+y^2}$, $(x,y) \neq (0,0)$ and $f(0,0) = 0$
2. Find an equation of tangent plane to the given surface at the specified point
   (a) $z = xe^{xy}$ at $P(1,0,1)$ (HW)
   (b) $z = y^2 - x^2$ at $P(1,1,0)$
   (c) $z = 3y^2 - x^2 - 3x$ at $P(2,-1,-7)$
3. Find $\frac{dz}{dt}$ in the following examples
   (a) $z = x^2+y^2;\ x = \cos t,\ y = \sin t$
   (b) $z = x^2+y^2;\ x = \cos t - \sin t,\ y = \cos t + \sin t$
4. Let $z = f(x,y);\ x = r\cos\theta,\ y = r\sin\theta$. Find $\frac{\partial z}{\partial r}, \frac{\partial z}{\partial \theta}$, and $\frac{\partial^2 z}{\partial \theta^2}$. (HW)
5. Find the directional derivative of the function $f(x,y) = x^3 - 3xy + 4y^2$ in the direction of unit vector $\mathbf{u} = \langle \cos\pi/6, \sin\pi/6\rangle$.
6. For $Y \in \mathbb{R}^3$ consider the function f defined by $f(X) = Y \cdot X,\ X = (x,y,z) \in \mathbb{R}^3$. Do directional derivatives of f exist in all directions? Is f differentiable at $(0,0,0)$.
7. Prove that if $f : \mathbb{R}^3 \to \mathbb{R}$ is differentiable at $X_0 = (x_0, y_0, z_0)$, then directional derivatives of f exist in all directions. (HW)
8. Consider the function defined by $f(x,y) = \frac{x^2y^2}{x^2+y^2}$, $(x,y) \neq (0,0)$ and $f(0,0) = 0$. Show that f is differentiable at $(0,0)$. (HW)